Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – discrete mathematics

Thursday 16 November 2017 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 11]

Mathilde delivers books to five libraries, A, B, C, D and E. She starts her deliveries at library D and travels to each of the other libraries once, before returning to library D. Mathilde wishes to keep her travelling distance to a minimum.

The weighted graph H, representing the distances, measured in kilometres, between the five libraries, has the following table.

	A	B	C	D	E
A	–	18	19	16	21
B	18	–	15	22	17
C	19	15	–	20	17
D	16	22	20	–	19
E	21	17	17	19	–

(a) Draw the weighted graph H. [2]

(b) Starting at library D use the nearest-neighbour algorithm, to find an upper bound for Mathilde's minimum travelling distance. Indicate clearly the order in which the edges are selected. [5]

(c) By first removing library C, use the deleted vertex algorithm, to find a lower bound for Mathilde's minimum travelling distance. [4]

2. [Maximum mark: 10]

Consider the recurrence relation

$$u_n = 5u_{n-1} - 6u_{n-2}\,,\ u_0 = 0 \text{ and } u_1 = 1\,.$$

(a) Find an expression for u_n in terms of n. [6]

(b) For every prime number $p > 3$, show that $p \mid u_{p-1}$. [4]

3. [Maximum mark: 11]

(a) (i) Draw the complete bipartite graph $\kappa_{3,3}$.

(ii) Prove that $\kappa_{3,3}$ is not planar. [5]

(b) A connected graph G has v vertices. Prove, using Euler's relation, that a spanning tree for G has $v - 1$ edges. [2]

Consider κ_n, a complete graph with n vertices, $n \geq 2$. Let T be a fixed spanning tree of κ_n.

(c) If an edge E is chosen at random from the edges of κ_n, show that the probability that E belongs to T is equal to $\frac{2}{n}$. [4]

4. [Maximum mark: 9]

Consider the system of linear congruences

$$x \equiv 2 \pmod 5$$
$$x \equiv 5 \pmod 8$$
$$x \equiv 1 \pmod 3.$$

(a) With reference to the integers 5, 8 and 3, state why the Chinese remainder theorem guarantees a unique solution modulo 120 to this system of linear congruences. [2]

(b) Hence or otherwise, find the general solution to the above system of linear congruences. [7]

5. [Maximum mark: 9]

(a) Convert the decimal number 1071 to base 12. [3]

(b) Write the decimal number 1071 as a product of its prime factors. [1]

The decimal number 1071 is equal to $a060$ in base b, where $a > 0$.

(c) (i) Using your answers to part (a) and (b), prove that there is only one possible value for b and state this value.

(ii) Hence state the value of a. [5]